2011 LSMSA Math Competition
Advanced Math Individual

1. How many positive integers less than 100 contain a 7 as a digit?

2. Evaluate $\cos(\sin(\tan(0)))$.

3. Find the height of an equilateral triangle of side length 1.

4. Evaluate $\sum_{i=-2010}^{2011} i$.

5. Find the sum of all factors of 72.

6. Find the loci of points equidistant from (1,0) and (0,1).

7. Assume the radius of the Earth is R. The Flash races Superman around the world. The Flash is arrogant and runs around the equator, while Superman runs around the 30° latitude line. The Flash runs at speed F, while Superman runs at $\frac{2}{3}F$. Who wins and what is the ratio of the loser's time to the winner's?

8. What is the probability of flipping a normal coin 5 times and getting 2 heads and 3 tails in any order?

9. A parabolic bridge spans 6 feet with a maximum height of 18 feet. Suddenly a flash flood causes the water to rise 10 feet above ground level, so the top of the bridge is only 8 feet above the water. What is the distance spanned by the bridge that is above water?

10. Evaluate $(1-i)^{24}$.

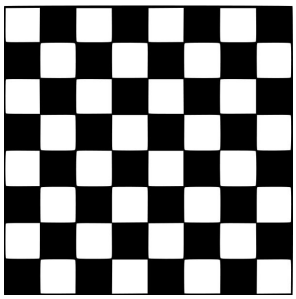
11. Mario is on a 1×1 cubic planet and wants to reach the princess on the opposite corner. How far must he move? Note that he can't go through the planet – he has to always be touching one of the cube's faces.

12. Find the next 5 in the sequence

$$4, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, \ldots$$

13. If $A + B = 7$ and $AB = 3$, find $A^2 + B^2$.

14. How many squares are on a chess board? (Remember to count the 2×2, 3×3, etc.)



15. Perform the sum $\sum_{n=1}^{\infty} \frac{n}{2^n}$.