2011 LSMSA Math Competition
Advanced Math Team

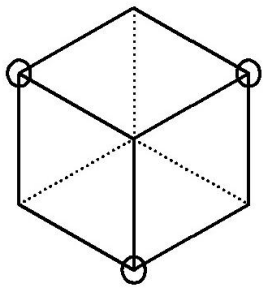
1. Find $\cos(3\theta)$ in terms of $\cos(\theta)$ and $\sin(\theta)$.

2. If $A+B+C+D+E=45$ and A, B, C, D, and E are in arithmetic progression ($B=r+A$, $C=r+B$, etc.), which of A, B, C, D, and E can you find and what is its value?

3. On 1B, there is a 0.4 probability that you'll hear a noise and, if there is a noise, there is a 0.9 probability that David Wolff is being reckless. If no noise is heard, there is a 0.4 probability that David is being reckless. If David's being reckless, what is the probability that noise is made?

4. Mario has legs of length 1 meter and takes one step per second, with each step creating a 30° angle between his legs. He eats a mushroom, changes color, and grows to 5x his original height (meaning his legs are now 5 times longer). Also, he takes longer steps so at the end of each step, there is a 60° angle between his legs. Still, he takes one step per second. What is the ratio of his new speed to his old speed?

5. A cube is going through a planar force-field. The three circled corners are in the plane of the force-field and the cross sectional area of the triangle found by these 3 points is $\frac{3\sqrt{3}}{4}$. Find the volume of the cube.



6. Find the Greatest Common Factor of all numbers in the form *abab* where a and b are digits. (For example 4343)

7. Evaluate $(\cos(15^\circ)-\sin(15^\circ))^{10}$.

8. A path of length $95\frac{1}{3}$ and width $1\frac{1}{2}$ spirals around and completely covers a rectangular garden. What are the dimensions of the garden? (The dimensions are integers)

9. A rectangular prism has an edge length of k and a surface area of m. In terms of k and m , what is the distance from one corner of the prism to the opposite corner?

10. Prove by induction: $\frac{n^5-n}{10}$ is an integer.

11. Write a formula that relates the prime factorization of any number to the sum of that number's factors. For example the sum of factors of 72 in terms of its prime factors: 2^3 and 3^2.

12. "How many children have you, and how old are they?" asked the guest, a mathematics teacher.

    "I have three boys," said Mr. Smith. "The product of their ages is 72 and the sum of their ages is the street number."

    The guest went to look at the entrance, came back and said:

    "The problem is indeterminate."

    "Yes, that is so," said Mr. Smith, "but I still hope that the oldest boy will some day win the Stanford competition."

    Tell the ages of the boys, stating your reasons.

13. Prove the identity

$$\cos\frac{\alpha}{2}\cos\frac{\alpha}{4}\cos\frac{\alpha}{8}=\frac{\sin\alpha}{8\sin\frac{\alpha}{8}}$$

14. Ten people are sitting around a round table. The sum of ten dollars is to be distributed among them according to the rule that each person receives one half of the sum that his two neighbors receive jointly. Is there just one way to distribute the money? Explain.

15. Solve the following Nurikabe Puzzle: