2011 LSMSA Math Competition
Calculus Individual

1. Find the equations of lines in $y = mx + b$ form if the lines pass through the point (2,4) and are tangent to the parabola $2x^2 - 3x + 10 = y$.

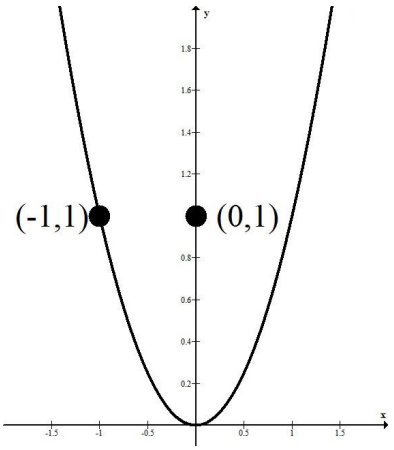
2. Evaluate $\lim_{n\to\infty} \frac{5n^6}{(5n+2)^7 - 5n^7}$.

3. Evaluate $\lim_{b\to 1^-} \int_0^b \frac{dx}{\sqrt{1-x^2}}$.

4. Evaluate $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right)$.

5. Evaluate $\frac{\frac{d^8}{dx^8}\left[x^8+x^7+x^6\right]}{\frac{d^6}{dx^6}\left[x^6+x^5+x^4\right]}$.

6. Nick Mead rides his bike along a parabolic valley, $y = x^2$, while looking at a babe at point (0,1). He moves with a constant x velocity, $\frac{dx}{dt} = 3$. He starts from the far left of the valley. At what rate, $\frac{d\theta}{dt}$, will he have to move his eyes to keep track of his Goddess's beauty when he is at (-1,1)? That is, assuming he doesn't fall.



7. Find $f'(2)$ if $f(x) = e^{g(x)}$ and $g(x) = \int_2^x \frac{t}{1+t^4}\, dt$.

8. Find $\int_{-50}^{50} \sin(\sin(x))\,dx$.

9. In terms of its height h and width w, find a formula for the area under a parabola.

10. Euler's gamma function $\Gamma(x)$ ("gamma of x"; Γ is a Greek capital g) uses an integral to extend the factorial function from the nonnegative integers to other real values. The formula is

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt, \qquad x > 0.$$

If n is a nonnegative integer, $\Gamma(n+1) = n!$

a. Show that $\Gamma(1) = 1$.

b. Then apply integration by parts to the integral for $\Gamma(x+1)$ to show that $\Gamma(x+1) = x\Gamma(x)$.

11. Find the slope of a line tangent to a circle of radius 2 centered at the origin at the point $(2, \theta)$ (in polar coordinates) in terms of θ.

12. Use Leibniz's Rule to find the value of x that maximizes the value of the integral

$$\int_x^{x+3} t(5-t)\,dt.$$

13. Evaluate $\lim_{x\to 0^+} (\cos(\sqrt{x})^{1/x}$.

14. Evaluate (in simplest form) $\frac{d}{dx}\left[\frac{\sin^3(x)}{\cos(x)} + \frac{1}{2}\left(\sin(2x)\right)\right]$.

15. a. If $\int_0^1 7f(x)\,dx = 7$, does $\int_0^1 f(x)\,dx = 1$?

b. If $\int_0^1 f(x)\,dx = 4$ and $f(x) \geq 0$, does $\int_0^1 \sqrt{f(x)}\,dx = \sqrt{4} = 2$?