Modern Algebra Homework

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March 11, 2010

1 Assignment

- Write a Caley table for the group $(S_3, \circ)$.
- Use every element to generate a subgroup to confirm its order.
- Find the inverse of every element

2 Solution

A group $(S_3, \circ) = (\{(1), (1,2), (1,3), (2,3), (1,2,3), (1,3,2)\}, \circ)$ is equal to

$$\left(\left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix},\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},\begin{pmatrix}1&3&2\\1&2&3\end{pmatrix},\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}\right\},\circ\right)$$

2.1 Caley Table

$\circ$	(1)	$(1,2)$	$(1,3)$	$(2,3)$	$(1,2,3)$	$(1,3,2)$
(1)	(1)	$(1,2)$	$(1,3)$	$(2,3)$	$(1,2,3)$	$(1,3,2)$
$(1,2)$	$(1,2)$	(1)	$(1,3,2)$	$(1,2,3)$	$(2,3)$	$(1,3)$
$(1,3)$	$(1,3)$	$(1,2,3)$	(1)	$(1,3,2)$	$(1,2)$	$(2,3)$
$(2,3)$	$(2,3)$	$(1,3,2)$	$(1,2,3)$	(1)	$(1,3)$	$(1,2)$
$(1,2,3)$	$(1,2,3)$	$(1,3)$	$(2,3)$	$(1,2)$	$(1,3,2)$	(1)
$(1,3,2)$	$(1,3,2)$	$(2,3)$	$(1,2)$	$(1,3)$	(1)	$(1,2,3)$

2.2 Subgroups

2.2.1 (1)

$$< (1) >= \{(1)\}$$

$C(\{(1)\}) = 1$. This means the order of (1) is 1.

2.2.2 (1,2)

$$< (1,2) >= \{(1,2), (1,2) \circ (1,2)\} = \{(1,2), (1)\}$$

$C(\{(1,2), (1)\}) = 2$. This means the order of $(1,2)$ is 2.

2.2.3 (1,3)

$$< (1,3) >= \{(1,3), (1,3) \circ (1,3)\} = \{(1,3), (1)\}$$

$C(\{(1,3), (1)\}) = 2$. This means the order of $(1,3)$ is 2.

2.2.4 (2,3)

$$< (2,3) >= \{(2,3), (2,3) \circ (2,3)\} = \{(2,3), (1)\}$$

$C(\{(2,3), (1)\}) = 2$. This means the order of $(2,3)$ is 2.

2.2.5 (1,2,3)

$$< (1,2,3) >= \{(1,2,3), (1,2,3) \circ (1,2,3), (1,2,3) \circ (1,2,3) \circ (1,2,3)\} = \{(1,2,3), (1,3,2), (1)\}$$

$C(\{(1,2,3), (1,3,2), (1)\}) = 3$. This means the order of $(1,2,3)$ is 3.

2.2.6 (1,3,2)

$$< (1,3,2) >= \{(1,3,2), (1,3,2) \circ (1,3,2), (1,3,2) \circ (1,3,2) \circ (1,3,2)\} = \{(1,3,2), (1,2,3), (1)\}$$

$C(\{(1,3,2), (1,3,2), (1)\}) = 3$. This means the order of $(1,3,2)$ is 3.

2.3 Inverses

From the table in Section 2.1, it is evident that the inverses of the elements are as follows:

Element	Inverse
(1)	(1)
$(1,2)$	$(1,2)$
$(1,3)$	$(1,3)$
$(2,3)$	$(2,3)$
$(1,2,3)$	$(1,3,2)$
$(1,3,2)$	$(1,2,3)$